ROTATIONAL MOTION AND ASTROPHYSICS

GRAVITATION

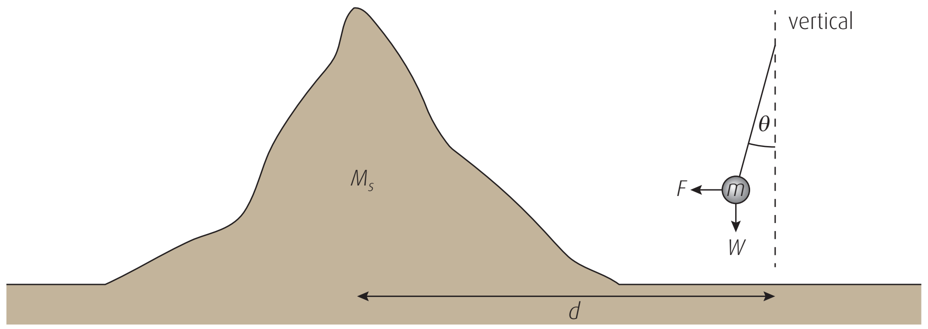
SCHIEHALLION EXPERIMENT

In 1774 Nevil Maskelyne led an expedition to the Perthshire mountain Schiehallion to measure the density of the Earth and also to confirm Newton's law of gravitation between non celestial masses on Earth.

Schiehallion has a symmetrical shape and is quite far away from other mountains which made it an ideal choice for the experiment.

Theory

A stationary pendulum held near the mountain would be deflected by a tiny amount, θ, towards the mountain due to the gravitational attraction between the mass of the mountain, M_s, and the mass of the pendulum bob, m.

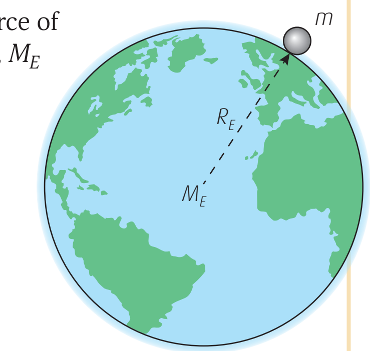


The gravitational force of attraction, F, between m and M_s is:

$$F = \frac{GmM_s}{d^2}$$

The weight, W, of the pendulum bob is the gravitational force of attraction between the mass, m, and the mass of the Earth, M_E

$$W = \frac{GmM_E}{R_E^2}$$



The ratio of F to W gives

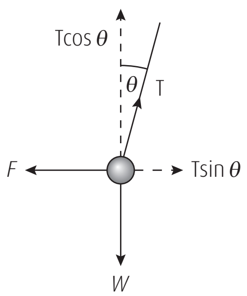
$$\frac{F}{W} = \frac{\frac{GmM_s}{d^2}}{\frac{GmM_E}{R_E^2}} = \frac{M_sR_E^2}{M_Ed^2}$$

Substituting M_s using the density of Schiehallion $\rho_s = \frac{M_s}{V_s}$

and also substituting M_E using the density of the Earth $\rho_E = \frac{M_E}{V_E}$ gives

$$\frac{F}{W} = \frac{\rho_sV_sR_E^2}{\rho_EV_Ed^2}$$

The tension T in the pendulum string can be split into components in the usual manner



$$F = T\sin\theta$$
$$W = T\cos\theta$$
$$\therefore \frac{F}{W} = \tan\theta$$

Substituting gives $\tan\theta = \frac{\rho_sV_sR_E^2}{\rho_EV_Ed^2}$

Finally rearranging $\rho_E = \frac{\rho_sV_sR_E^2}{V_Ed^2 \times \tan\theta}$ where the six quantities on the right hand side can all be measured.

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DON'T FORGET

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

Taking the measurements

θ was the most difficult measurement as it was so small. Maskelyne was the Astronomer Royal and he used accurate astronomical instruments, clocks and his knowledge of star positions to measure his current latitude and then the vertical direction at this latitude. Next he measured the direction of the stationary pendulum string. The difference between vertical and the pendulum string direction was θ. This was carried out at two locations north and south of Schiehallion to reduce uncertainties. His value of θ was 11.6 arc seconds. An angle of 1° can be subdivided up into 60 arc minutes or 3600 arc seconds. So 11.6 arc seconds is 3.2×10^{-3} degrees.

The volume of Schiehallion was measured by the mathematician and surveyor Charles Hutton. He took thousands of survey points and developed the system of contour lines of equal height to work out the volume of individual sections before adding them up.

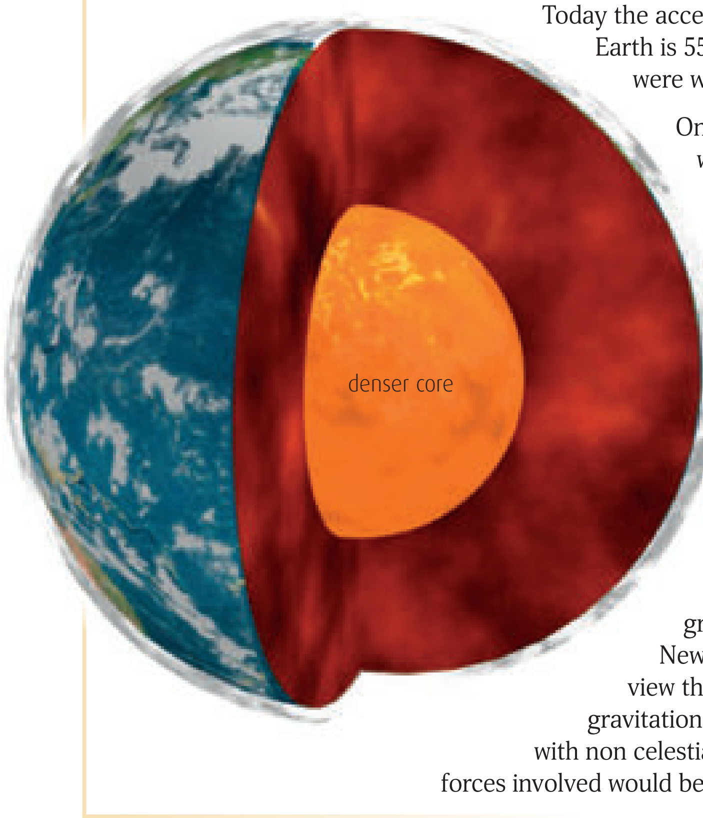
Charles Hutton also estimated the distance d from the pendulum bob to the centre of the mountain using all his survey points.

The density of Schiehallion was found using rock samples and dividing mass by volume. The average value was around 2500 kg m^{-3} (in today's metric units).

The radius and volume of the Earth were known in 1774 as the Earth's circumference had been measured quite accurately by different methods over preceding centuries.

Average density of the Earth

Maskelyne's experiment gave a value for the density of the Earth of 4500 kg m^{-3} which was almost twice the density of the mountain. This led to the hypothesis that the core of the Earth must be denser than the crust and must be metallic as metals are more dense than stone and rock.



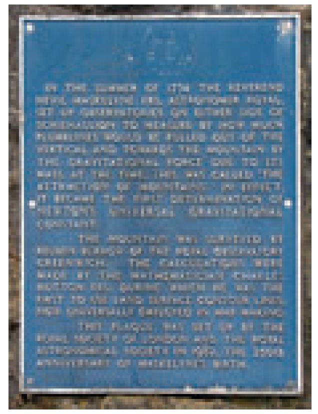
Today the accepted value of the density of the Earth is 5514 kg m^{-3} so Maskelyne's results were within 20% of the accepted value.

Once the density of the Earth was found then the mass of the Earth could be found as well ($m = \rho \times V$). This was the real prize as up to this point the masses of the Sun, planets and their moons could only be stated as a relative ratio with the Earth's mass. Now they could be calculated individually. Maskelyne's experiment is often referred to as "weighing the Earth".

This experiment also confirmed experimentally Newton's law of gravitation from 100 years earlier. Newton at the time had expressed the view that experimental verification of gravitational attraction would not be possible with non celestial masses as the gravitational forces involved would be too small to measure.

ONLINE

Follow the link at www.brightredbooks.net for an excellent historical background to the experiment and also the difficulties faced by Maskelyne and his team.



Plaque at Schiehallion car park

THINGS TO DO AND THINK ABOUT

The mean value of the Earth's radius is 6371 km and the Earth's mean density is 5514 kg m^{-3}. Use these values to find the mean mass of the Earth. Why should your answer for the Earth's mass have four significant figures when using this data?

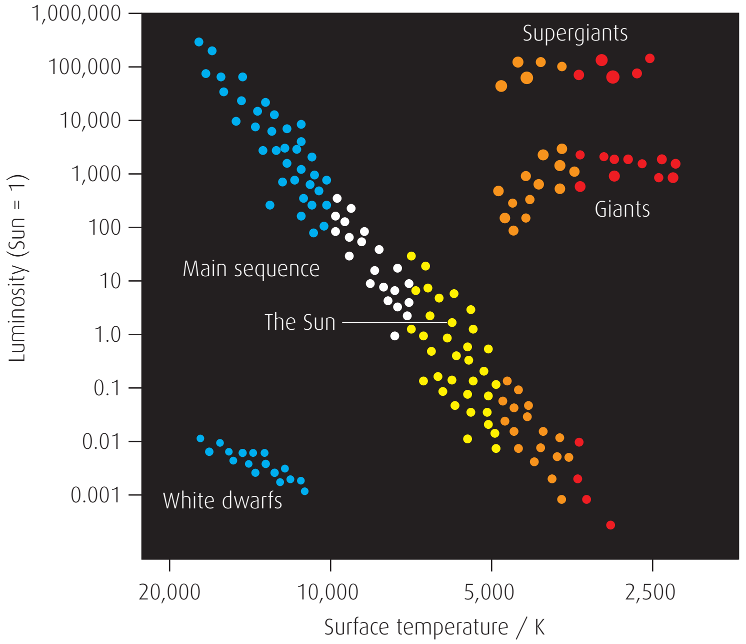
ONLINE TEST

Test yourself on this topic online at www.brightredbooks.net

STELLAR PHYSICS

HERTZSPRUNG-RUSSELL (H-R) DIAGRAM

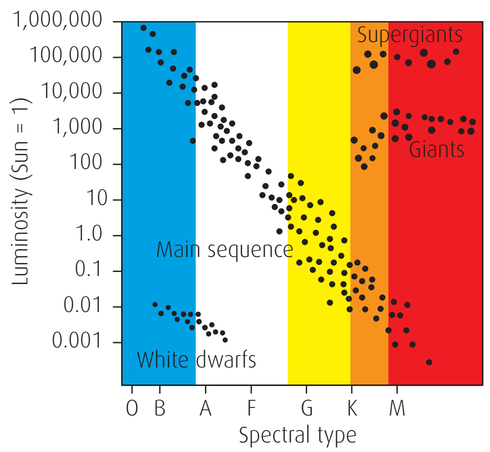
The H-R diagram is a scatter graph that plots a star's luminosity against its surface temperature. When hundreds of known stars are entered on the H-R diagram the results are not a random scatter but fall into four distinct groups.



Note the temperature axis scale increases from right to left rather than the more conventional left to right and the axes scales are not linear.

Main sequence stars account for 90% of the stars in the universe and these stars fuse hydrogen nuclei into helium nuclei. The Sun is a main sequence star with coordinates (5600,1) on the above diagram.

Giant stars and supergiant stars have much bigger radii and luminosities than main sequence stars of the same surface temperature and are found in the top right of the H-R diagram. All the hydrogen in the core has been fused into helium and the helium in turn now fuses into carbon.



White dwarfs are the final evolutionary state of most stars once fusion has stopped. The very hot core of the former giant star has no source of energy and will gradually cool as it radiates heat energy.

More detail on the evolution of stars is given on pages **.

The original H-R diagram was created by Enjar Hertzsprung and Henry Russell working independently around 1911–12. Hertzsprung's horizontal axis used the colour of light given off by each star. Russell used letters identifying the spectral type of the light given off by each star. The letters O, B, A, F, G, K and M were used which are meaningful and useful to astronomers.

Surface temperature of the star is perhaps more user friendly for our purposes.

The H-R diagram opposite shows how Spectral Type and the colour of light emitted can be used as alternatives to Star Surface Temperature.

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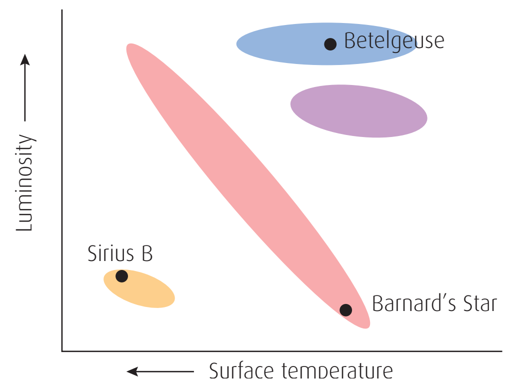
Comparing stars with the same surface temperature

Three named stars are shown on the following H-R diagram.



Betelgeuse and Barnard's star have approximately the same surface temperature. It follows that they must have similar power outputs per square metre of surface area.

But Betelgeuse is much more luminous than Barnard's star so it must have a greater surface area to account for this greater overall power output. Betelgeuse is a much bigger star with more surface area than Barnard's star.

Comparing stars with the same luminosity

Barnard's star and Sirius B have similar (low) luminosities. But Sirius B is much hotter with a greater surface temperature. The power output per unit surface area of Sirius B is much greater than that of Barnard's star so Sirius B must have a much smaller surface area to account for this.

Sirius B has a much smaller radius and smaller surface area than Barnard's star.

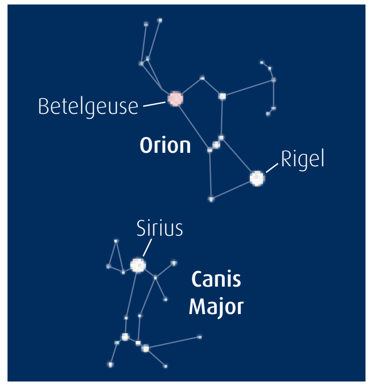
Sirius A

Sirius A is the brightest star in the sky seen from Earth and is a main sequence star with surface temperature of 9940 K and luminosity 25 times greater than the Sun ($25L_\odot$).

Sirius A is usually just called Sirius as its near neighbour Sirius B is so dim that it cannot be seen by the naked eye from Earth.

Sirius is called the "dog star" as it is part of the Canis Major (great dog) constellation just below Orion (the hunter).

Sirius can be found by approximately projecting the line of Orion's belt to the lower left.



Sirius is relatively close to Earth at a distance of 8.6 ly which also contributes to it brightest star status.

In ancient Roman times Sirius could still be seen in the sky at sunrise between mid July and mid August. The expression "dog days of summer" (meaning sultry and oppressive hot weather) comes from the time when the dog star could still be seen as the sun was rising.

DON'T FORGET

Stefan-Boltzman relationship for power output P per unit area $P = \sigma T^4$, see page **.

This photo was taken from the Hubble telescope showing Sirius B as a faint dot to the lower left of Sirius A. The photograph of Sirius A was overexposed to enable Sirius B to be seen.

ACTIVITY:

1 In which region of the H-R diagram would you find

  a a hot star with a low luminosity (a hot dim star)

  b a cool star with high luminosity (a cool bright star)?

2 A star is hotter than the Sun and with a higher luminosity. What can be deduced about the radius of the star compared to the radius of the Sun?

ONLINE

For interactive questions on the H-R diagram, go to www.brightredbooks.net

THINGS TO DO AND THINK ABOUT

Red dwarf stars form a subdivision of the main sequence at the bottom right of the H-R diagram. They are cooler, dimmer, smaller and less massive than the Sun. Proxima Centauri, the nearest star to the Sun, is a red dwarf. Red dwarf stars are predicted to live for trillions of years before turning into blue dwarfs. Blue dwarf stars are hypothetical at the present time and none have been observed. Can you think why none have been found yet?

ONLINE TEST

Head to www.brightredbooks.net and test yourself on this topic.