BGE

COURSE BOOK

Level 3 MATHEMATICS

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BGE LEVEL 3

MATHEMATICS

COURSE BOOK

COVERS THE MATHEMATICS CURRICULUM • FROM S1 TO S3 •

NUMERACY SKILLS

MULTIPLES, FACTORS AND PRIMES

Common multiples and factors

I have investigated strategies for identifying common multiples and common factors, explaining my ideas to others, and can apply my understanding to solve related problems. MTH 3-05a

What's coming up?

This Outcome and Experience will give you the opportunity to:

- identify common multiples, including the lowest common multiple for whole numbers and explain the method used
- identify common factors, including the highest common factor for whole numbers and explain the method used
- solve problems using multiples and factors.

What you already know

You have already learned how to:

- ✔ identify multiples and factors of whole numbers
- ✔ apply knowledge and understanding of these when solving relevant problems in number, money and measurement.

Why do we need to learn how to find multiples, lowest common multiples (LCMs), or highest common factors (HCFs)?

Easy! So we can do what mathematics is all about: solving problems!

Common multiples

How does that work?

Multiples are simply your 'times tables'.

For example:

Write down the first twelve multiples of	3
They are	3, **6**, 9, **12**, 15, **18**, 21, **24**, 27, 30, 33, 36

Write down the first twelve multiples of	2
They are	2, 4, **6**, 8, 10, **12**, 14, 16, **18**, 20, 22, 24
Do you notice any multiples in common?	6, 12, 18, 24
What is the lowest number in this list?	6
6 is the **lowest common multiple (LCM)** of	2 and 3

The lowest common multiple is useful when working out real-life problems.

How does that work?

EXAMPLE

Katrina is having a barbeque.
She goes to the shop to buy burgers and buns.
The burgers come in packs of 6, but the buns come in packs of 8.
What is the least number of burgers and buns Katrina could buy to make sure there are no 'extras' or waste?

SOLUTION

So, Katrina could have:

burgers	6	12	18	**24**	30	36	...
buns	8	16	**24**	32	40	48	...

Katrina sees that 24 is the lowest common multiple (LCM).

So she should buy	4 packs of burgers	= 4 × 6	= 24 burgers
	3 packs of buns	= 3 × 8	= 24 buns

That is, each bun has a burger and there are no leftover buns or burgers!

How does that work?

The lowest common multiple is also really useful when adding, or subtracting, 'unlike' fractions

We can 'multiply up' to give a common denominator.

For example,

$\frac{1}{2} + \frac{1}{3}$ The LCM of 2 and 3 is 6 — Multiples of 2: 2, 4, **6**, 8, 10 ...; Multiples of 3: 3, **6**, 9, 12 ...

$= \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

We will revisit this later in the book, in the 'fractions' section.

Classroom challenge

1 Write down
- a the first twelve multiples of 4
- b the first nine multiples of 3
- c the first eight multiples of 5
- d the first ten multiples of 10.

2 Write down
- a all the multiples of 4 between 31 and 61
- b all the multiples of 9 between 26 and 96
- c all the multiples of 7 between 40 and 100
- d all the multiples of 6 between 50 and 110.

3 a Write down the first fifteen multiples of 2.
b What is the name given to these numbers?

4 21, 24, 27, 30, 33, 36 could be described as 'the multiples of 3 from 21 to 36'.
Describe the following lists of numbers in the same way.
- a 15, 20, 25, 30, 35, 40
- b 60, 66, 72, 78, 84
- c 36, 45, 54, 63, 72, 81, 90
- d 70, 80, 90, 100, 110, 120
- e 60, 80, 100, 120, 140
- f 42, 45, 48, 51, 54

5 Find the LCM of the following pairs of numbers.
- a 3 and 4
- b 2 and 5
- c 6 and 9
- d 4 and 10
- e 8 and 7
- f 6 and 5
- g 7 and 3
- h 5 and 12
- i 7 and 11

DON'T FORGET

You may wish to write out the first few multiples of the larger number – and look for the first number in the list into which the smaller number divides exactly

6 a Write down the first ten multiples of 2.
b Write down the first eight multiples of 3.
c Write down the first six multiples of 9.
d Write down the lowest common multiple (LCM) of 2, 3 and 9.

7 Using a method similar to that used above, find the LCM of
- a 2, 3 and 5
- b 2, 5 and 6
- c 3, 4 and 8
- d 2, 3, and 4
- e 4, 8 and 12
- f 3, 4 and 20
- g 5, 10 and 30
- h 3, 6 and 12
- i 4, 7 and 14

8 Graeme is a lighthouse keeper.
He sets the light to flash every 3 minutes.
He sets the horn to sound every 7 minutes.
At 9pm, both the light flashes and the horn sounds.
When will they next flash and sound together?

DON'T FORGET

You should consider writing out the multiples of 3 and then the multiples of 7. Is there a common multiple? What is the LCM?

9 In the United States I can vote for a president every 4 years in a ballot.
I can vote for the Senate every 6 years in a ballot.
In 2018, I voted for both the President and the Senate in the same ballot.
In what year will I, again, be able to vote for both of these positions in the same ballot?

10 The number 3 tram takes 24 minutes to complete its route and return to the terminus
The number 7 tram takes 18 minutes to complete its route and return to the terminus.
Both trams leave the terminus at 8am.
When will they next be at the terminus together?

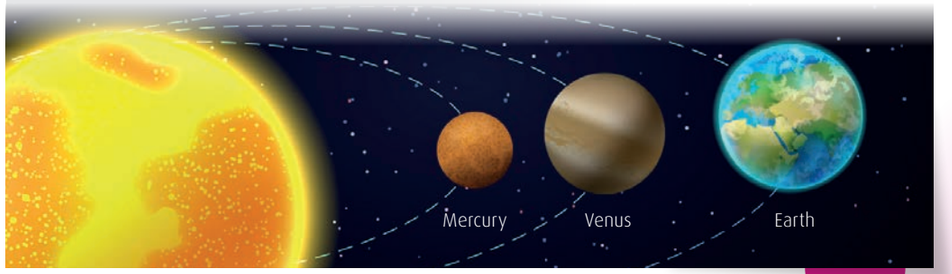
11 Marcus is setting up his Christmas lights.
The red lights flash every 6 seconds
The blue lights flash every 9 seconds
The yellow lights flash every 10 seconds.
Marcus notices that all three lights have flashed together.
After how many seconds will the lights flash together again?

12 Jenny pays her telephone bill every 8 weeks.
She pays her electricity bill every 9 weeks.
She pays her gas bill every 6 weeks.
Jenny notices that she paid them all on 1st June
After how many weeks will she next pay them all at the same time?

13 Callum, Grigor and Rory live in Australia and 'Skype' their mother in Scotland.
Callum Skypes every 8 days
Grigor Skypes every 5 days
Rory Skypes every 6 days.
Today, all three Skyped their mother.
How many days will it be until all three skype on the same day again?

STRETCH YOURSELF

14 The Earth takes 1 year to orbit the Sun.
Venus takes $\frac{2}{3}$ of a year to orbit the Sun.
If the two planets are in alignment, as shown in the diagram, how long will it be before they return to this position?



STRETCH YOURSELF

15 Now, let's add in Mercury!
Mercury takes $\frac{1}{4}$ of a year to orbit the Sun.
Remember: Earth takes 1 year
Venus takes $\frac{2}{3}$ of a year.
All three are in alignment with the Sun.
How many years will it take for all three to return to this alignment?

DON'T FORGET

Think about multiples of 1 and multiples of $\frac{2}{3}$!

Common factors

Remember that factors are simply numbers that divide exactly into another number.

For example, the factors of 6 are 1, 2, 3, 6.

How does that work?

EXAMPLE

Write down all the factors of 12.

DON'T FORGET

The factors of a number will always include 1 and the number itself.

SOLUTION

It is best to work in 'number pairs' to find factors

12 = 1 × 12
2 × 6
3 × 4

We can stop here as the next 'pair' would be 4 × 3 which is same as 3 × 4.
The factors of 12 are 1, 2, 3, 4, 6, 12.

Like multiples, factors are often used in fraction calculations to simplify the fraction.

Common factors also appear in algebra.

However, common factors, and especially the highest common factor (HCF), are often used to solve problems.

How does that work?

EXAMPLE

Clarke is catering for a party.
He has 72 sticks of celery and 48 carrot sticks.
He wants both foods to be on each plate.
He wants to share the food out evenly and have no waste.
What is the largest number of plates he can make up?
How much of each food would be on each plate?

SOLUTION

Write down all the factors of 72. Write down all the factors of 48.
When writing down factors it is best to write them in pairs – so that you don't miss any out!

72		
1	72	1 × 72
2	36	2 × 36
3	24	3 × 24
4	18	4 × 18
6	12	6 × 12
8	9	8 × 9

48		
1	48	1 × 48
2	24	2 × 24
3	16	3 × 16
4	12	4 × 12
6	8	6 × 8

It is now easy to list the factors, in order, for each number.
Factors of 72 are **1**, **2**, **3**, **4**, **6**, **8**, 9, **12**, 18, **24**, 36, 72
Factors of 48 are **1**, **2**, **3**, **4**, **6**, **8**, **12**, 16, **24**, 48
Factors common to both lists are **1**, **2**, **3**, **4**, **6**, **8**, **12**, **24**
The highest number in this list is 24
The **highest common factor (HCF)** of 72 and 48 is **24**
This means that the greatest number of plates Clarke could have is **24**
For food on each plate: celery sticks 72 ÷ **24** = 3 so 3 sticks of celery on each plate.
carrot sticks 48 ÷ **24** = 2 so 2 sticks of carrot on each plate.

Classroom challenge

1 Write down all four factors of 8.
2 Write down all six factors of 18. Put them in order from smallest to largest.
3 The number 10 has four factors. List them in order, smallest to largest.
4 Write down all three factors of 25.
5 Write down all the factors of 36.
6 Using 'the pairs method' write down all the factors of

a 7	d 22	g 32	j 45
b 14	e 40	h 100	k 23
c 24	f 29	i I 54	l 60

7 What do you notice about the number of factors for each part of question 6?
8 Write down all the factors of

a 1	d 16	g 49	j 100
b 4	e 25	h 64	k 121
c 9	f 36	i I 81	l 144

9 a. What do you notice about the number of factors for each part of question7?
b. What do we call this type of number? (1, 4, 9, 16, 25, 36 ... ...)

10 Write down all the factors of

a	2	d	7	g	17	j	29
b	3	e	11	h	19	k	31
c	5	f	13	i	I 23	l	37

11 a. What do you notice about the number of factors for each part of question 9?
b. Do you know what these numbers are called?
We will look at them more closely in the next section.

12 a. List all the factors of 12, in order from smallest to largest.
b. List all the factors of 8, in order from smallest to largest.
c. Write all the common factors; that is the numbers which appear in both lists.
d. What is the highest common factor (HCF) of 12 and 18?

13 a. List, in order, all the factors of 15.
b. List, in order, all the factors of 20.
c. What is the HCF of 15 and 20?

14 Find the highest common factor (HCF) for each of the following pairs of numbers.

a	6 and 8	c	12 and 24	e	15 and 60	g	19 and 57
b	30 and 50	d	24 and 36	f	8 and 36	h	18 and 24

15 Alicia is working for her school charities group.
She has 36 pencils and 24 pens which she is putting into pencil cases to send to children in need.
She wants to share the pencils and pens equally, and does not want any left over.
a What is the maximum number of pencil cases Alicia could fill?
b How many pencils and pens would go into each pencil case?

DON'T FORGET
Look back at the example.

16 Elton works in a florist shop.
He has 48 red tulips and 32 white tulips.
He wants to share the red and white tulips equally in vases.
He does not want to have any tulips left over.
a What is the maximum number of vases Elton could make?
b How many of each colour of tulip would go into each vase?

17 George is trying to encourage his friends to use public transport.
He has 18 bus tickets and 12 tram tickets
He wants to make up envelopes to give to his friends.
He wants the envelopes to contain both bus and tram tickets, and for these to be shared equally.
He does not want any tickets left over.
a What is the maximum number of envelopes George could fill?
b How many bus tickets and how many tram tickets would be in each envelope?

18 Lorna is making bracelets using beads.
She has 45 green beads and 36 blue beads.
She wants to share the green and blue beads equally on the bracelets
She does not want to have any beads left over.
a What is the maximum number of bracelets Lorna could make?
b How many of each colour of bead would be on each bracelet?

STRETCH YOURSELF

19 Lorna has bought in new stock of beads.
She now has 120 green beads
90 blue beads
45 white beads.
She wants to share the 3 colours on all the bracelets.
She does not want any beads left over.
a What is the maximum number of bracelets Lorna could make?
b How many of each colour of bead would be on each bracelet?

Ready for an investigation?
6 is a perfect number
Why?
List the factors of 6 (except 6 itself)
1, 2, 3
Add them up
1 + 2 + 3 = 6!
If the factors of a number add up to that number, then it is called a perfect number.
Can you find some more?

Working with prime numbers

I can apply my understanding of factors to investigate and identify when a number is prime. MTH 3-05b

What's coming up?

This Outcome and Experience will give you the opportunity to:
- identify prime numbers to 100 and explain the method used
- write a given number as a product of its prime factors.

What you already know

You have already learned how to:
- ✔ identify multiples and factors of whole numbers
- ✔ apply knowledge and understanding of these when solving relevant problems in number, money and measurement.

ALGEBRAIC SKILLS

PATTERNS AND RELATIONSHIPS

Having explored number sequences, I can establish the set of numbers generated by a given rule and determine a rule for a given sequence, expressing it using appropriate notation. MTH 3-13a

What's coming up?

This Outcome and Experience will give you the opportunity to:
- generate number sequences from a given rule, for example, $t = 4n + 6$
- extend a given pattern and describe the rule
- express sequence rules in algebraic notation, for example, the cost of hiring a car is £75 plus a charge of £0·05 per mile, 'm' driven, $C = 0{\cdot}05m + 75$.

What you already know

You have already learned how to:
- ✔ explain and use a rule to extend well known number sequences including square numbers, triangular numbers and Fibonacci sequence
- ✔ apply knowledge of multiples, square numbers and triangular numbers to generate number patterns.

Patterns and sequences

Some number patterns are familiar, and some may take a little more thought!

How does that work?

A familiar number pattern is 2, 4, 6, 8, 10, 12, ...
A 'rule' for this pattern could be 'even numbers starting at 2'
or it could be 'start at 2 and go up in twos'.

Another familiar pattern is 1, 3, 5, 7, 9, 11, ...
A 'rule' for this pattern could be 'odd numbers starting at 1'
or it could be 'start at 1 and go up in twos'.

A **sequence** has a **starting point** and a **rule** to find the **next term**.
The starting point is usually called the **1st term**.
Each **term** in a **sequence** is made by following the **same rule**.

EXAMPLE

Starting point	4
Rule	add 3 (+3)
Sequence	4, 7, 10, 13, 16, 19, ...

EXAMPLE

1st term	1
Rule	double (×2)
Sequence	1, 2, 4, 8, 16, 18, ...

EXAMPLE

1st term	60
Rule	subtract 5 (– 5)
Sequence	60, 55, 50, 45, 40, 35, ...

Classroom challenge

1 For each row in the table, create a sequence using the 1st term and the given rule.
For each one, write down the first 6 terms.
The first one has been done for you.

1st term	Rule	Sequence
1	Add 3	1, 4, 7, 10, 13, 16, ...
3	Double	
4	Add 6	
2	Multiply by 5	
80	Subtract 5	
10	Multiply by 10	
6	Add 6	

2 Write down the next 2 terms for each of these sequences.
Also write down the rule you used to get these terms.
a 2, 6, 10, 14, 18, 22, ..., ...
b 7, 15, 23, 31, 39, ..., ...
c 90, 87, 84, 81, 78, ..., ...
d 4, 12, 36, 108, 324, ..., ...
e 88, 77 66, 55, 44, ..., ...
f 1, 5, 25, 125, 625, ..., ...
g 81, 72, 63, 54, ..., ...

3 Write down the next 2 terms of these sequences.
Write down the rule you used to get these terms.
a 5, 10, 15, 20, ..., ...
b 4, 8, 12, 16, ..., ...
c 9, 18, 27, 36, ..., ...
d 10, 20, 30, 40, ..., ...
e 3, 6, 9, 12, ..., ...
f 2, 4, 6, 8, 10, ..., ...
g 7, 14, 21, 28, ..., ...

Look at question 3a above.

Term	1	2	3	4	5	6
Value	5	10	15	20	25	30

5 5 5 5 5

The values go up in 5s. Exactly like the 5 times table!

5 × **1** = 5 gives **1**st term
5 × **2** = 10 gives **2**nd term
5 × **3** = 15 gives **3**rd term
5 × **4** = 20 gives **4**th term

This suggests that the **5**th term will be found by doing
5 × **5** = 25 which it is!
This suggests that the **6**th term will be found by doing
5 × **6** = 30 which it is!
It also shows a rule to find any term.
For example, to find the **100**th term simply multiply by 5
5 × **100** = 500
So, to find the '***n***th' term simply multiply by 5
5 × ***n*** = 5*n*
This gives us a rule for finding any term in the sequence
*n*th term = 5*n*

4 a What will the 7th term be?
b What will the 10th term be?
c What will the 1000th term be?

5 Copy and complete this table for the sequence in question 3b above.

Term	1	2	3	4	5	6
Value	4	8	12	16		

a What will the 7th term be?
b What will the 10th term be?
c What will the 100th term be?
d What will the *n*th term be?

6 Construct a table for each of the other sequences in question 3.
Complete the rules for finding the '*n*th' term.
a *n*th term = × *n* =
b *n*th term = × *n* =
c *n*th term = × *n*
d *n*th term
e *n*th term

7 Use the rule given to write down the first 5 terms of each sequence.
The first one has been done for you.
a 3*n* + 2 *n* = 1, gives 3 × 1 + 2 = 5, *n* = 2, gives 3 × 2 + 2 = 8, *n* = 3 gives 3 × 3 + 3 = 11
n = 4, gives 3 × 4 + 2 = 14, *n* = 5 gives 3 × 5 + 2 = 17
First 5 terms are 5, 8, 11, 14, 17
b 4n + 1
c 5n + 3
d 6n – 1
e 7n
f 8n – 3
g 10n – 2
h 12n
i 11n
j 3n – 2

Sequences and algebraic notation

Sometimes a sequence may be generated by using a rule or a formula.

How does that work?

John is a plumber.
When he is called out on an emergency he charges a £50 call-out fee plus £20 per hour for the time taken to fix the problem.
Construct a formula to show John's charge.
Let's call the charge *C* and the number of hours taken to be *h*.
The charge is made up of a fixed fee of £50 and £20 × the number of hours

That is $C = £50 + £20 \times h$

The formula for calculating the charge is $C = 50 + 20h$

This can now be used to calculate a charge for any number of hours taken.

Number of hours (*h*)	1	2	3	4	5	6	7
Calculation of charge	50 + 20 × 1	50 + 20 × 2	50 + 20 × 3	50 + 20 × 4	50 + 20 × 5	50 + 20 × 6	50 + 20 × 7
'Sequence' £	70	90	110	130	150	170	190

If John spent 10 hours on an emergency repair, how much should he charge?

Formula C = 50 + 20h
Substitute = 50 + 20 × 10
Calculate = 50 + 200
C = 250
John should charge £250 for the job.

Classroom challenge

8 Kelly is an emergency engineer.
She charges a call-out fee of £60 plus £30 per hour when doing an emergency repair.
a Construct a formula for Kelly's charge.
Let the charge be *C* and the number of hours worked be *h*.
Copy and complete:
The charge is made up of a fixed fee of and × number of hours
C = £ + £ ×
C =
b Copy and complete this table to show Kelly's charge for different numbers of hours.

Number of hours (*h*)	1	2	3	4	5	6	7
Calculation of charge	60 + 30 × 1						
'Sequence' £	90						

9 A window cleaner charges £2 to visit a house and then £0·50 for every window cleaned.
   a Write down a formula for the window cleaner's total charge. Use C for charge and w for number of windows.
   b How much would the window cleaner charge if he visited a house with
      i 5 windows?
      ii 10 windows?
      iii 15 windows?

10 A taxi driver charges a flat fee of £4 plus £1·20 per mile travelled.
   a Write down a formula for the taxi driver's charge. Use C for charge and m for miles.
   b Jane takes the taxi from work to home, a distance of 8 miles. How much would she be charged?

11 Neptune chocolate bars cost 32p each.
Write down a formula for the cost, C pence, of n Neptune bars.

12 Petrol costs £1·33 per litre.
Write down a formula for the cost, C pounds, of l litres of petrol.

13 A maths tutor charges £25 per hour for lessons and a fixed fee of £12 for materials.
   a How much would she charge for 3 lessons.
   b Write down a formula to calculate the cost of l lessons. Use C = ...
   c How much would she charge for (i) 5, (ii) 10, (iii) 15 lessons?

14 Dave and Carol saw this advert for hiring a narrowboat.
   a Write down a formula for the cost (C) of hiring the narrowboat if you know the number of days d.
   b Copy and complete this table for costs for different number of days hire.

Number of days (d)	1	2	3	4	5	6	7
Calculation of charge	100 + 1 × 45						
'Sequence' £	90						

   c How much would it cost Dave and Carol to hire the narrowboat for 12 days?

15 Bert's Bicycle holidays charges £65 per day plus a fixed charge of £25 for the hire of a bike and a helmet.
   a Make up a table to show the Number of days (d) and the Total cost (C) for up to 5 days hire.
   b Write down a formula for calculating the cost of d days hire.
   c How much would it cost to hire a bike from Bert's for (i) 7 days (ii) 14 days?

16 Azim is an electrician. He charges a call-out fee of £50 and £85 per hour.
   a Construct a formula for Azim's charge
   b How much would Azim charge for 5 hours work?

STRETCH YOURSELF

   c If Azim charged a customer £390, how many hours had he worked?