NUMBER, MONEY AND MEASURE

ALGEBRAIC SKILLS

PATTERNS AND RELATIONSHIPS

Having explored number sequences, I can establish the set of numbers generated by a given rule and determine a rule for a given sequence, expressing it using appropriate notation. MTH 3–13a

What's coming up?

This Outcome and Experience will give you the opportunity to:
- generate number sequences from a given rule, for example, $t = 4n + 6$
- extend a given pattern and describe the rule
- express sequence rules in algebraic notation, for example, the cost of hiring a car is £75 plus a charge of £0·05 per mile, 'm' driven, $C = 0·05m + 75$.

What you already know

You have already learned how to:
✔ explain and use a rule to extend well known number sequences including square numbers, triangular numbers and Fibonacci sequence
✔ apply knowledge of multiples, square numbers and triangular numbers to generate number patterns.

Patterns and sequences

Some number patterns are familiar, and some may take a little more thought!

How does that work?

A familiar number pattern is   2, 4, 6, 8, 10, 12, … .
A 'rule' for this pattern could be   'even numbers starting at 2'
or it could be   'start at 2 and go up in twos'.

Another familiar pattern is   1, 3, 5, 7, 9, 11, … .
A 'rule' for this pattern could be   'odd numbers starting at 1'
or it could be   'start at 1 and go up in twos'.

A **sequence** has a **starting point** and a **rule** to find the **next term**.
The starting point is usually called the **1st term**.
Each **term** in a **sequence** is made by following the **same rule**.

EXAMPLE

Starting point	4
Rule	add 3 (+3)
Sequence	4, 7, 10, 13, 16, 19, … …

EXAMPLE

1st term	1
Rule	double (×2)
Sequence	1, 2, 4, 8, 16, 18, … …

EXAMPLE

1st term	60
Rule	subtract 5 (– 5)
Sequence	60, 55, 50, 45, 40, 35, … …

Classroom challenge

1 For each row in the table, create a sequence using the 1st term and the given rule.
   For each one, write down the first 6 terms.
   The first one has been done for you.

1st term	Rule	Sequence
1	Add 3	1, 4, 7, 10, 13, 16, … …
3	Double	
4	Add 6	
2	Multiply by 5	
80	Subtract 5	
10	Multiply by 10	
6	Add 6	

2 Write down the next 2 terms for each of these sequences.
   Also write down the rule you used to get these terms.
   a   2, 6, 10, 14, 18, 22, …, …   d   4, 12, 36, 108, 324, …, …   f   1, 5, 25, 125, 625, …, …
   b   7, 15, 23, 31, 39, …, …   e   88, 77 66, 55, 44, …, …   g   81, 72, 63, 54, …, …
   c   90, 87, 84, 81, 78, …, …

3 Write down the next 2 terms of these sequences.
   Write down the rule you used to get these terms.
   a   5, 10, 15, 20, …, …   d   10, 20, 30, 40, …, …   f   2, 4, 6, 8, 10, …, …
   b   4, 8, 12, 16, …, …   e   3, 6, 9, 12, …, …   g   7, 14, 21, 28, …, …
   c   9, 18, 27, 36, …, …

Look at question 3a above.

Term	1	2	3	4	5	6
Value	5	10	15	20	25	30

5   5   5   5   5

The values go up in 5s. Exactly like the 5 times table!

5 × **1** = 5   gives   **1**st term
5 × **2** = 10   gives   **2**nd term
5 × **3** = 15   gives   **3**rd term
5 × **4** = 20   gives   **4**th term

This suggests that the **5**th term will be found by doing
5 × **5** = 25   which it is!
This suggests that the **6**th term will be found by doing
5 × **6** = 30   which it is!
It also shows a rule to find any term.
For example, to find the **100**th term simply multiply by 5
5 × **100** = 500
So, to find the '***n***th' term simply multiply by 5
5 × ***n*** = 5*n*
This gives us a rule for finding any term in the sequence
*n*th term = 5*n*

4 a   What will the 7th term be?
   b   What will the 10th term be?
   c   What will the 1000th term be?

5 Copy and complete this table for the sequence in question 3b above.

Term	1	2	3	4	5	6
Value	4	8	12	16		

   a   What will the 7th term be?   c   What will the 100th term be?
   b   What will the 10th term be?   d   What will the *n*th term be?

6 Construct a table for each of the other sequences in question 3.
   Complete the rules for finding the '***n***th' term.
   a   *n*th term = × *n* =   d   *n*th term
   b   *n*th term = × *n* =   e   *n*th term
   c   *n*th term = × *n*

7 Use the rule given to write down the first 5 terms of each sequence.
   The first one has been done for you.
   a   3n + 2   *n* = 1, gives 3 × 1 + 2 = 5, *n* = 2, gives 3 × 2 + 2 = 8, *n* = 3 gives 3 × 3 + 3 = 11
                    n = 4, gives 3 × 4 + 2 = 14, *n* = 5 gives 3 × 5 + 2 = 17
                    First 5 terms are 5, 8, 11, 14, 17

   b   4n + 1   e   7n   h   12n
   c   5n + 3   f   8n – 3   i   11n
   d   6n – 1   g   10n – 2   j   3n – 2

Sequences and algebraic notation

Sometimes a sequence may be generated by using a rule or a formula.

How does that work?

John is a plumber.
When he is called out on an emergency he charges a £50 call-out fee plus £20 per hour for the time taken to fix the problem.
Construct a formula to show John's charge.
Let's call the charge *C* and the number of hours taken to be *h*.
The charge is made up of a fixed fee of £50 and £20 × the number of hours

That is *C* = £50 + £20 × *h*

The formula for calculating the charge is *C* = 50 + 20*h*

This can now be used to calculate a charge for any number of hours taken.

Number of hours (*h*)	1	2	3	4	5	6	7
Calculation of charge	50 + 20 × 1	50 + 20 × 2	50 + 20 × 3	50 + 20 × 4	50 + 20 × 5	50 + 20 × 6	50 + 20 × 7
'Sequence' £	70	90	110	130	150	170	190

If John spent 10 hours on an emergency repair, how much should he charge?

Formula   C   = 50 + 20h
Substitute   = 50 + 20 × 10
Calculate   = 50 + 200
                     C   = 250
John should charge   £250 for the job.

Classroom challenge

8 Kelly is an emergency engineer.
   She charges a call-out fee of £60 plus £30 per hour when doing an emergency repair.
   a   Construct a formula for Kelly's charge.
        Let the charge be *C* and the number of hours worked be *h*.
        Copy and complete:
        The charge is made up of   a fixed fee of   and   × number of hours
                                      C =   £   +   £   ×
                                      C =
   b   Copy and complete this table to show Kelly's charge for different numbers of hours.

Number of hours (*h*)	1	2	3	4	5	6	7
Calculation of charge	60 + 30 × 1						
'Sequence' £	90						

9   A window cleaner charges £2 to visit a house and then £0·50 for every window cleaned.
    a   Write down a formula for the window cleaner's total charge. Use *C* for charge and *w* for number of windows.
    b   How much would the window cleaner charge if he visited a house with
        i   5 windows?
        ii   10 windows?
        iii 15 windows?

10 A taxi driver charges a flat fee of £4 plus £1·20 per mile travelled.
    a   Write down a formula for the taxi driver's charge. Use *C* for charge and *m* for miles.
    b   Jane takes the taxi from work to home, a distance of 8 miles. How much would she be charged?

11 Neptune chocolate bars cost 32p each.
    Write down a formula for the cost, *C* pence, of *n* Neptune bars.

12 Petrol costs £1·33 per litre.
    Write down a formula for the cost, *C* pounds, of *l* litres of petrol.

13 A maths tutor charges £25 per hour for lessons and a fixed fee of £12 for materials.
    a   How much would she charge for 3 lessons.
    b   Write down a formula to calculate the cost of *l* lessons. Use *C* = ...
    c   How much would she charge for (i) 5, (ii) 10, (iii) 15 lessons?

Narrowboat Hire £100 + £45 per day

14 Dave and Carol saw this advert for hiring a narrowboat.
    a   Write down a formula for the cost (*C*) of hiring the narrowboat if you know the number of days *d*.
    b   Copy and complete this table for costs for different number of days hire.

Number of days (*d*)	1	2	3	4	5	6	7
Calculation of charge	100 + 1 × 45						
'Sequence' £	90						

    c   How much would it cost Dave and Carol to hire the narrowboat for 12 days?

15 Bert's Bicycle holidays charges £65 per day plus a fixed charge of £25 for the hire of a bike and a helmet.
    a   Make up a table to show the Number of days (*d*) and the Total cost (*C*) for up to 5 days hire.
    b   Write down a formula for calculating the cost of *d* days hire.
    c   How much would it cost to hire a bike from Bert's for (i) 7 days (ii) 14 days?

16 Azim is an electrician. He charges a call-out fee of £50 and £85 per hour.
    a   Construct a formula for Azim's charge
    b   How much would Azim charge for 5 hours work?

STRETCH YOURSELF

    c   If Azim charged a customer £390, how many hours had he worked?