Substitution

As simple as replacing a letter with a number

Substitution in maths is really just the same as when you use a substitute in sport, but instead of replacing one player for another you replace a letter with a number.

Example 1

$B = 3a^2 - c^2$

Calculate B when $a = 2$ and $c = -5$

Solution

$B = 3 \times 2^2 - (-5)^2$ ◗ Substitute in values.

$= 3 \times 4 - 25$ ◗ Evaluate each part – take care with the negatives.

$= 12 - 25$ ◗ Work out your answer.

$= -13$

Example 2

$f(x) = 10 - 3x$

(a) Evaluate $f(-4)$.

(b) Given that $f(a) = 11$, find a.

Solution

(a) $f(-4) = 10 - (3 \times -4)$ ◗ Substitute in values.

$= 10 - (-12)$ ◗ Evaluate each part – take care with the negatives.

$= 10 + 12$ ◗ Work out your answer.

$= 22$

(b) $f(a) = 10 - 3a$ ◗ Substitute a into $f(x)$.

$10 - 3a = 11$ ◗ Set the 2 expressions for $f(a)$ equal to each other.

$-3a = 1$ ◗ Solve the equation to find a.

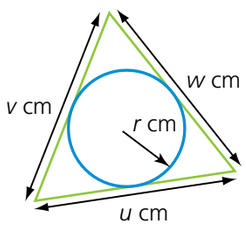
$a = -\frac{1}{3}$

Formulae

Evaluating a formula

When evaluating a formula it is essential that you remember the order of operations (see BODMAS pg **).

Example 1

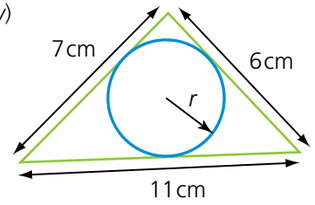


A circle can be drawn inside a triangle as shown in the diagram on the left.

The radius, r centimetres, of this circle can be found using the formula

$r = \sqrt{\frac{(s-u)(s-v)(s-w)}{s}}$, where $s = \frac{1}{2}(u + v + w)$

Use this formula to find the radius of the circle shown in the diagram on the right.



continued

Evaluating a formula – continued

Solution

$u = 11 \quad v = 7 \quad w = 6$ ◗ State the values of u, v and w.

$s = \frac{1}{2}(u + v + w)$ ◗ State the formula for s.

$= \frac{1}{2}(11 + 7 + 6)$ ◗ Substitute into the formula for s.

$= \frac{1}{2}(24)$ ◗ Calculate s.

$= 12$

$r = \sqrt{\frac{(s-u)(s-v)(s-w)}{s}}$ ◗ State the formula for r.

$= \sqrt{\frac{(12-11)(12-7)(12-6)}{12}}$ ◗ Substitute in the values.

$= \sqrt{\left(\frac{30}{12}\right)}$ ◗ Evaluate the numerator and denominator. Put a bracket around $\frac{30}{12}$ so that you square root the whole fraction and not just 30.

$= 1.58\,\text{cm}$ ◗ Calculate r.

Example 2

Kirsty has a pay and go mobile phone but pays a fixed price of £10 per month for 200 text messages. If she exceeds 200 texts in a month she is then charged 12 pence per message.

(a) How much does Kirsty pay in a month when she sends 235 texts?

(b) Write down a formula for the total amount, £T, Kirsty pays in a month when the number of texts, t, she sends is greater than 200.

Solution

(a) Total cost $= 10 + 0.12(235 - 200)$ ◗ £10 is charged every month. 200 minutes are free so we subtract them from the total number of texts sent.

$= 10 + 4.2$ ◗ Multiply the number of texts being charged for by £0.12

$= £14.20$ ◗ Calculate the answer – remember to include the units.

(b) $T = 10 + 0.12(t - 200)$ ◗ We want a formula for the total amount so we know that $T =$. £10 is a standard charge each month. £0.12 is charged on each text that Kirsty pays for. $(t - 200)$ works out how many texts Kirsty pays for.

$T = 10 + 0.12t - 24$ ◗ Multiply out the brackets and simplify.

$T = 0.12t - 14$

(Check: $T = 0.12 \times 235 - 14 = £14.20$ which is the same as above) ◗ Check that the formula works for 235 texts.

Multiplying expressions

Be careful with negatives!

When you are asked to simplify an expression be careful with any negatives. Remember that the negative on the outside of a bracket multiplies **everything** inside it.

Look out for

$-(x + 2)$ means that you multiply everything inside the bracket by -1.

Example 1

Simplify $2(3 - 4x) - 3(5 - 4x)$.

Solution

$2(3 - 4x) - 3(5 - 4x)$

$= 6 - 8x - 15 + 12x$ ◗ Multiply out each bracket. Notice the second bracket is multiplied by -3.

$= -9 + 4x$ ◗ Collect like terms.

$= 4x - 9$ ◗ The line in blue is not needed but looks better!

Example 2

Expand and simplify $(x - 1)(x^2 + 3x - 10)$.

Solution

$(x - 1)(x^2 + 3x - 10)$

$= x^3 + 3x^3 - 10x - x^2 - 3x + 10$ ◗ Multiply everything in the second bracket by x: $x^3 + 3x^3 - 10x$
Multiply everything in the second bracket by -1: $-x^2 - 3x + 10$

$= x^3 + 2x^2 - 13x + 10$ ◗ Collect like terms.

◗ The brackets can also be answered using a table if you find it easier.

	x^2	$+ 3x$	$- 10$
x	x^3	$+ 3x^2$	$- 10x$
-1	$-x^2$	$- 3x$	$+ 10$

Algebraic fractions

Basic rules

The rules for working with algebraic fractions are exactly the same as those for numerical fractions.

We must always have a common denominator to add or subtract fractions. The easiest way to do this is to multiply fraction 1 by the denominator of fraction 2 and vice versa.

Just like numerical fractions, algebraic fractions should always be given in their simplest form. Factorising the numerator and denominator, when possible, will allow you to see if you can simplify.

eg $\frac{12 + 2x^2}{8x}$

$= \frac{2(6 + 2x^2)}{8x}$

$= \frac{6 + 2x^2}{4x}$

Look out for

Always give your answer over 1 denominator and make sure the line covers everything that is on the top and bottom.

eg $\frac{3}{2x} + \frac{x}{4}$

$= \frac{4 \times 3}{4 \times 2x} + \frac{2x \times x}{2x \times 4}$

$= \frac{12}{8x} + \frac{2x^2}{8x}$

$= \frac{12 + 2x^2}{8x}$

Example 1

Simplify $\frac{4}{x - 1} - \frac{2}{x}$

Solution

$\frac{4}{x - 1} - \frac{2}{x} = \frac{4x}{x(x - 1)} - \frac{2(x - 1)}{x(x - 1)}$ ◗ Get a common denominator by multiplying top and bottom by the opposite denominator.

$= \frac{4x - 2(x - 1)}{x(x - 1)}$ ◗ Put over 1 denominator.

$= \frac{4x - 2x + 2}{x(x - 1)}$ ◗ Expand the bracket on the top line. Be careful of the negative!

$= \frac{2x + 2}{x(x - 1)}$ ◗ Simplify the numerator.

$= \frac{2(x + 1)}{x(x - 1)}$ ◗ Factorise the top line, if possible simplify.