DIFFERENTIATION

You should know all of the differentials in the following table or be able to derive them, as follows:

for example, $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$ and then use the quotient rule (or the product rule) as shown below.

DON'T FORGET

e^{3x} can be written as $\exp(3x)$

DERIVATIVES TO LEARN

$f(x)$	$f'(x)$	Unit
e^x	e^x	[Unit 1]
$\ln x$ (or $\log_e x$)	$\frac{1}{x}$	[Unit 1]
$\tan x$	$\sec^2 x$	[Unit 1]
$\sec x$	$\sec x \tan x$	[Unit 1]
$\text{cosec } x$	$-\frac{\cos x}{\sin^2 x} = -\cot x \text{ cosec } x$	[Unit 1]
$\cot x$	$-\text{cosec}^2 x$	[Unit 1]
$\sin^{-1} x$, $\sin^{-1} ax$	$\frac{1}{\sqrt{1-x^2}}$, $\frac{a}{\sqrt{1-(ax)^2}}$	[Unit 2]
$\cos^{-1} x$, $\cos^{-1} ax$	$\frac{-1}{\sqrt{1-x^2}}$, $\frac{-a}{\sqrt{1-(ax)^2}}$	[Unit 2]
$\tan^{-1} x$, $\tan^{-1} ax$	$\frac{1}{1+x^2}$, $\frac{a}{1+(ax)^2}$	[Unit 2]

DON'T FORGET

$f'g + fg'$

DON'T FORGET

You should be able to recognise and use different notations:

i) functional: $f(x)$, $f'(x)$, $f''(x)$

ii) Leibniz: $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$

PRODUCT RULE: Unit 1

$$(f(x)\,g(x))' = f'(x)\,g(x) + f(x)g'(x)$$

Example 1.1

$$\frac{d}{dx}(4x \sin x) = 4 \sin x + 4x \cos x$$

DON'T FORGET

Remember to square the denominator! $[g(x)]^2$

DON'T FORGET

$\frac{f'g - fg'}{g^2}$

QUOTIENT RULE: Unit 1

$$\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$$

Example 1.2

$$\frac{d}{dx}\left(\frac{\sin x}{\ln x}\right) = \frac{\cos x \ln x - \sin x.\frac{1}{x}}{(\ln x)^2} = \frac{x \cos x \ln x - \sin x}{x (\ln x)^2}$$

DON'T FORGET

Remember that you always need to tidy up your solutions!

CHAIN RULE: Unit 1

$$(f(g(x)))' = f'(g(x)).g'(x)$$

Example 1.3

$$\frac{d}{dx}(e^{\tan x}) = e^{\tan x}.\sec^2 x$$

Example 1.4

$$\frac{d}{dx}(\tan^{-1}(1-2x)) = \frac{1}{1+(1-2x)^2} \times (-2) = \frac{-2}{2-4x+4x^2} = -\frac{1}{1-2x+2x^2}$$

DON'T FORGET

Remember, $\frac{dy}{dx} = \frac{1}{dx/dy}$

(Unit 1)

Remember also:

$f'(x) = \lim_{x\to 0}\left(\frac{f(x+h)-f(h)}{h}\right)$

(Unit 1)

HIGHER DERIVATIVES: Unit 1

$$\frac{d^{n+1}y}{dx^{n+1}} = \frac{d}{dx}\left(\frac{d^n y}{dx^n}\right)$$

Example 1.5

$$y = (3x+1)^4$$

$$\frac{dy}{dx} = 12(3x+1)^3$$

$$\frac{d^2y}{dx^2} = 108(3x+1)^2$$

Example 1.6

$$\frac{d^2y}{dx^2} = \sin^4 2x$$

$$\frac{d^3y}{dx^3} = 4(\sin^3 2x).2\cos 2x$$

$$\frac{d^3y}{dx^3} = 8\cos 2x \sin^3 2x$$

DON'T FORGET

Give exact values in your answers, rather than decimal approximations.

DON'T FORGET

Be aware that any of these differentiation skills could be tested in contextualised questions, such as *optimisation* or *displacement/velocity/acceleration*.

For more useful notes on Differentiation visit http://www.hsn.uk.net/resources/HSN28000

LET'S THINK ABOUT THIS

Past Paper Questions (solutions can be found on page xx.)

2004 Q1

(a) Given $f(x) = \cos^2 x\, e^{\tan x}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, obtain $f'(x)$ and evaluate $f'\left(\frac{\pi}{4}\right)$. 3, 1

(b) Differentiate $g(x) = \frac{\tan^{-1} 2x}{1+4x^2}$ 3

2005 Q1

(a) Given $f(x) = x^3 \tan 2x$, where $0 < x < \frac{\pi}{4}$, obtain $f'(x)$. 3

(b) For $y = \frac{1+x^2}{1+x}$, where $x \neq -1$, *determine* $\frac{dy}{dx}$ in simplified form. 3

2006 Q10

The amount x micrograms of an impurity removed per kg of a substance by a chemical process depends on the temperature T °C as follows:

$x = T^3 - 90T^2 + 2400T$, $10 \leq T \leq 60$.

At what temperature in the given range should the process be carried out to remove as much impurity per kg as possible? 4